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*Rapid Note*

# Discrete scale invariance in viscous fingering patterns

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**Abstract.** We study viscous fingering patterns in a lifting Hele-Shaw cell, where a non-Newtonian fluid (oil paint) is displaced by air. The lengths of the air fingers are measured and their cumulative distribution is seen to follow a power law with log-periodic oscillations indicating the presence of discrete scale invariance.

**PACS.** 47.20.Ma Interfacial instability – 83.10.Ji Fluid dynamics (nonlinear fluids) – 68.10.-m Fluid surfaces and fluid–fluid interfaces

Viscous fingering (VF) is a wellknown example of interfacial instability in Laplacian growth [1].

The prototype model for producing and studying VF is the Hele-Shaw cell [2]. The cell consists of two parallel glass plates separated by a small distance ($\sim$ mm). A viscous fluid occupies the space between the plates. In the radial geometry Hele-Shaw cell a less viscous fluid is forced under pressure into the more viscous fluid, through a hole in the centre of the upper plate. The interface between the two fluids becomes unstable and a pattern of finger-like intrusions is formed, whose characteristics depend on conditions such as viscosity of the fluids, interface tension and forcing pressure.

A variation of the conventional Hele-Shaw cell is the lifting Hele-Shaw cell (LHSC) [3].

Patterns in the LHSC where a non-Newtonian fluid (oil paint) is displaced by air was studied by Tarafdar and Roy [4], Roy and Tarafdar [5].

We discuss our experimental set-up for LHSC in some detail. The cell consists of two circular glass plates of about 10 cm diameter. The upper plate can be raised or lowered by a pneumatic cylinder arrangement. The two plates are always parallel.

To generate the pattern, a blob of oil-paint is placed at the centre of the lower plate, and the upper plate is lowered to press down upon it. The paint spreads out with a more or less circular outline. Now the upper plate is raised slowly and air enters the gap, in the form of fingers, displacing the paint. A characteristic pattern is produced as shown in Figure 1.

It is necessary for the viscosity contrast between the displaced and displacing fluids to be high, for the interface to become unstable and form the fingering pattern. If the displacing fluid has a high viscosity comparable to the displaced fluid, the paint surface retracts retaining a smooth circular perimeter. We found this to be the case with pure glycerine displacing the paint.

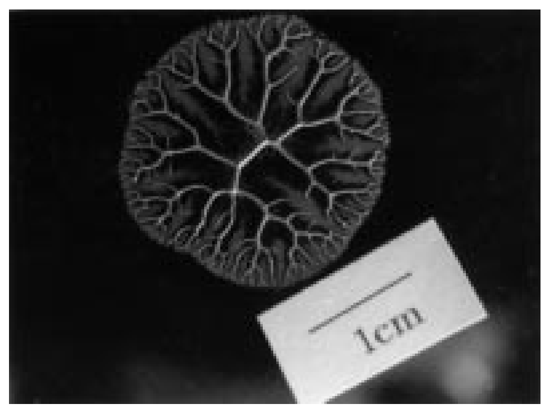


**Fig. 1.** A pattern formed by air displacing oil paint in the lifting Hele-shaw cell.

Another term sometimes used for this type of modification of the Hele-Shaw cell is "Variable Hele-Shaw cell" or VHSC. In a slightly different arrangement, the upper plate is lifted from one end, using the other end as a pivot. In this case the gap between the plates is wedge-shaped and changes with distance from the pivot, as well as with time, during lifting. This method of separation was used in [3] and [6], but the arrangement is called LHSC in [3], while [6] refers to it as VHSC, so the two terms are used almost synonymously. In this work we call our system LHSC as this term appears more appropriate for the parallel lifting

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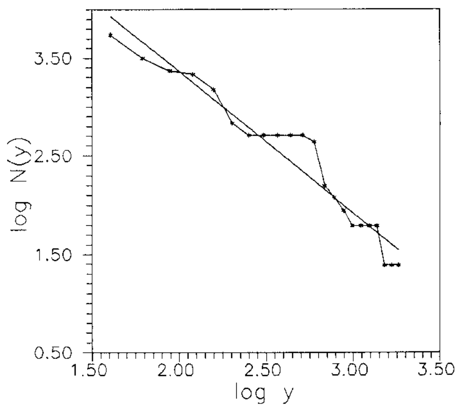


**Fig. 2.** Log$N(y)$ plotted against $\log y$ shows oscillations. log denotes the natural logarithm. The straight line has a slope $-1.44$.

model. In [5] both methods of lifting have been used but the cell was referred to as VHSC.

In our patterns the most striking difference from the conventional Hele-Shaw cell pattern is that finally the *displaced fluid* (paint) forms the tree like structure rather than the displacing fluid. The air fingers which enter at the sides constitute the background. Fractal dimensions of approximately 1.5 were found for the tree pattern by box counting .

It has recently been shown [7] that contrary to previous ideas, naturally formed patterns may exhibit *discrete* rather than continuous scale invariance. Whereas for mathematical deterministic fractals scale invariance is exact for a definite dilation factor $\lambda$ and its integral powers only, random fractals found in nature are expected to be invariant for continuously varying length scales. If a *deterministic* pattern is measured on a continuously varying length scale and the results plotted on a log-log graph, one expects a stepped or oscillatory variation decorating a linear power-law behaviour. But several natural systems such as seismic activity records, DLA growth, a system of growing cracks etc. exhibit periodic oscillations on a log-log plot of their characteristic scale invariant quantities. This is a signature of discrete scale invariance (DSI) governing the natural process. It implies the existence of an underlying dilation factor which is a characteristic of the process, in addition to the Hausdorff dimension which describes the average power-law behaviour. It is discussed in [7] that such systems can be equivalently described by a complex fractal dimension, where the dilation factor $\lambda$ determines the imaginary part. If $f$ is the frequency of the log-periodic oscillation

$$f = 1/\log\lambda$$

where "log" is the natural logarithm.

The object of the present work is to demonstrate that viscous fingering also belongs to the family of natural processes which show DSI. It was discussed in [5] that the paint pattern formed on displacing by air is a hierarchical pattern very similar to several other patterns found in nature and is very regular – almost deterministic. Other systems where this pattern is found are river basin boundary geometry [8] and growth of crystals within the pore space in rocks [9].

**Table 1.** Results for the 5 patterns studied.

| Pattern no. | exponent $m$ | $\lambda$ from periodogram |
|---|---|---|
| 1 | $-1.44$ | 1.248 |
| | | 1.162 |
| 2 | $-1.22$ | 1.488 |
| 3 | $-1.36$ | 1.546 |
| | | 1.469 |
| 4 | $-1.29$ | 1.088 |
| | | 1.193 |
| 5 | $-1.31$ | 1.434 |
| | | 1.147 |

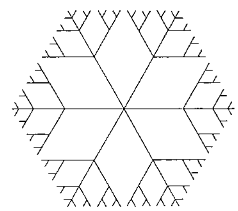

**Fig. 3.** An idealization of the pattern in Figure 1.

In the present communication we focus on the air fingers, rather than the paint pattern. We analyse the length distribution of the air fingers and find that they follow an approximate power law with log-periodic oscillations characteristic of discrete scale invariance (DSI) discussed by Sornette [7]. Five patterns have been generated and studied, the results of analysis of the patterns is presented.

Figure 1 shows a typical pattern produced in our LHSC. The air fingers are seen to enter at the sides displacing the oil paint to form the pattern. The lengths of the air fingers were measured ignoring any branching and $\log N(y)$ is plotted against $\log y$ in Figure 2. Here $N(y)$ is the number of fingers with length $\geq y$. Periodic oscillations are observed about the average linear fit. The smallest and largest fingers have been omitted, these deviate from the average linear behaviour. A deterministic idealized version of Figure 1 is shown in Figure 3. The cumulative finger length distribution of Figure 3 will obviously show discrete steps, which are present with some amount of noise in the real pattern (Fig. 1). The average linear behaviour indicates a power law

$$N(y) \sim y^{-m}.$$

To demonstrate more clearly the oscillations in Figure 2, we calculated the local derivative of $\log N(y)$ with respect to $\log y$. A typical result is plotted in Figure 4.

A Lomb periodogram of this result is shown in Figure 5. We have constructed a histogram of the periodogram results for the five patterns studied, this is shown in Figure 6. In Table 1 we show the results of our analysis of the five patterns. From the most prominent peaks in the histogram of the Lomb periodogram we get $\lambda = 1.47$ and 1.25. This is considerably different from the universal value of $\lambda = 2$ suggested in references [7,10]. A detailed

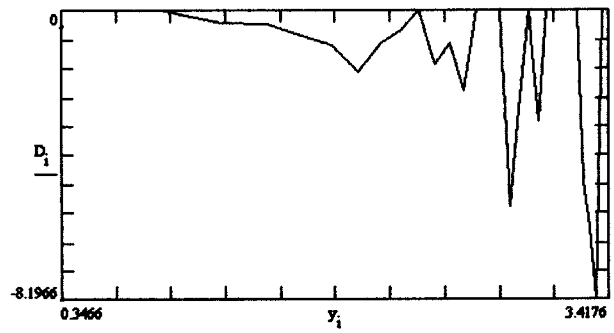


**Fig. 4.** $\mathrm{d\,Log}N(y)/\mathrm{d}\log y$ plotted against log$y$ shows the periodic oscillations more clearly.

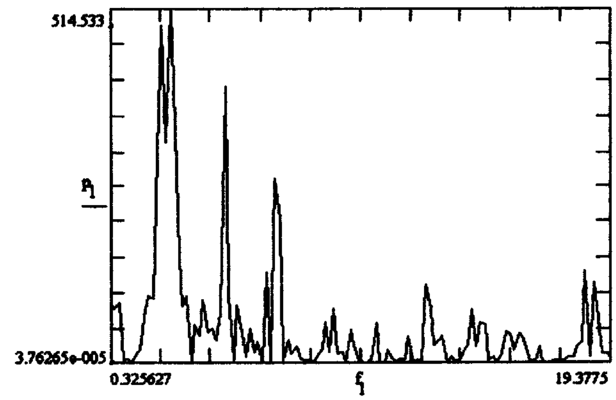


**Fig. 5.** Lomb periodogram of the data shown in Figure 4.

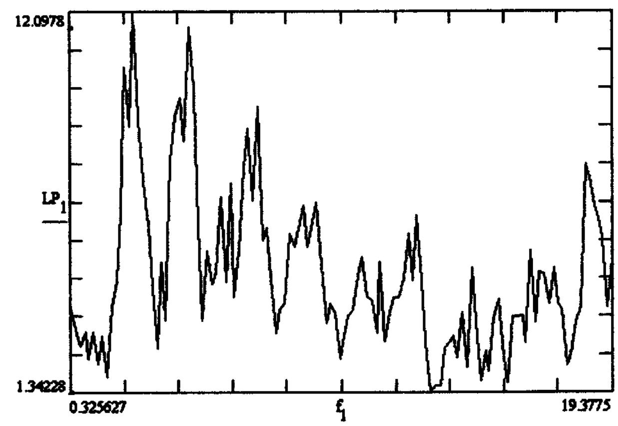


**Fig. 6.** Histogram of the periodograms for the five patterns studied.

analysis of the process of pattern formation may shed some light on the reason for this.

The periodograms are rather noisy with a large number of small peaks, this is probably because we have used raw data without any smoothing filter for the results shown in Figures 2 and 4. We shall try to improve our method of analysis in future work. However, this preliminary investigation shows interesting results, and the presence of DSI seems quite clear. It is interesting to compare our results with the work presented by Huang *et al.* [11], on DSI in a system of cracks.

The finger length distribution in our experiment is very similar to the results of the model suggested by Huang *et al.* [11]. The growth process suggested there is as follows. If $n$ fingers start growing initially, after a certain time interval every alternate finger is suppressed. This causes a period doubling of the initial growth mode. The process is repeated regularly so that the number of growing fingers is halved at definite intervals. Our LHSC pattern seems to follow exactly this mechanism of growth. We have plans to study the time development of the fingers by videophotography to see if the dynamics agrees with the model suggested by Huang *et al.* [11].

In earlier work on air-paint patterns a fractal dimension for the tree like paint pattern was obtained. Non-Newtonian nature of the fluids is usually said to incorporate non-linearity in the system and produce fractal patterns [12]. In our earlier work [5], the patterns were manually produced and looked less symmetric, there was in addition significant branching of the air fingers, this resulted in a fractal pattern. The patterns produced in the new mechanically driven LHSC are closer to the ideal noise free pattern of Figure 3, particularly for small sizes of the order of $\sim 1$ cm radius. In this case the paint pattern has no non-integral fractal dimension. However the air finger distribution gives a scale invariant behaviour with DSI which underlines the inherent deterministic character of the pattern.

Larger patterns produced with more paint are not as regular and symmetric. They give a fractal dimension of $\sim 1.8$ on box counting.

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